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Mark Scheme (Results)

Summer 2013

GCE Core Mathematics 3 (6665/01R)

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

EDEXCEL GCE MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 75.

2. The Edexcel Mathematics mark schemes use the following types of marks:

- **M** marks: method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
- **A** marks: accuracy marks can only be awarded if the relevant method (M) marks have been earned.
- **B** marks are unconditional accuracy marks (independent of M marks)
- Marks should not be subdivided.

3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes:

- bod – benefit of doubt
- ft – follow through
- the symbol $\sqrt{}$ will be used for correct ft
- cao – correct answer only
- cso - correct solution only. There must be no errors in this part of the question to obtain this mark
- isw – ignore subsequent working
- awrt – answers which round to
- SC: special case
- oe – or equivalent (and appropriate)
- dep – dependent
- indep – independent
- dp decimal places
- sf significant figures
- $\ast$ The answer is printed on the paper
- The second mark is dependent on gaining the first mark

4. All A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.

5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

6. If a candidate makes more than one attempt at any question:
   - If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
   - If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.

7. Ignore wrong working or incorrect statements following a correct answer.

8. In some instances, the mark distributions (e.g. M1, B1 and A1) printed on the candidate's response may differ from the final mark scheme.

General Principles for Core Mathematics Marking

(But note that specific mark schemes may sometimes override these general principles).

Method mark for solving 3 term quadratic:

1. Factorisation

$$(x^2+bx+c)=(x+p)(x+q), \text{ where } |pq|=|c|, \text{ leading to x =}$$

$$(ax^2+bx+c)=(mx+p)(nx+q), \text{ where } |pq|=|c| \text{ and } |mn|=|a|, \text{ leading to x =}$$

2. Formula

Attempt to use correct formula (with values for *a*, *b* and *c*).

3. Completing the square

$$\text{Solving } x^2+bx+c=0: \quad \left(x\pm\frac{b}{2}\right)^2 \pm q \pm c, \quad q\neq 0, \quad \text{leading to x =...}$$

Method marks for differentiation and integration:

1. Differentiation

Power of at least one term decreased by 1. ($x^n \to x^{n-1}$)

2. Integration

Power of at least one term increased by 1. ($x^n \to x^{n+1}$)

Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.
Normal marking procedure is as follows:
Method mark for quoting a correct formula and attempting to use it, even if there are mistakes in the substitution of values.
Where the formula is not quoted, the method mark can be gained by implication from correct working with values, but may be lost if there is any mistake in the working.

Exact answers

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

Answers without working

The rubric says that these may not gain full credit. Individual mark schemes will give details of what happens in particular cases. General policy is that if it could be done "in your head", detailed working would not be required.

Question Number	Scheme	Marks
1.	(a) $x^2+x-12=(x+4)(x-3)$	B1
	Attempt as a single fraction $\dfrac{(3x+5)(x-3)-2(x^2+x-12)}{(x^2+x-12)(x-3)}$ or $\dfrac{3x+5-2(x+4)}{(x+4)(x-3)}$	M1
	$=\dfrac{x-3}{(x+4)(x-3)}$, $=\dfrac{1}{(x+4)}$ cao	A1, A1
		(4 marks)

Notes for Question 1

B1 For correctly factorising $x^2+x-12=(x+4)(x-3)$. It could appear anywhere in their solution

M1 For an attempt to combine two fractions. The denominator must be correct for 'their' fractions.
The terms could be separate but one term must have been modified.
Condone invisible brackets.
Examples of work scoring this mark are;

$\dfrac{(3x+5)(x-3)}{(x^2+x-12)(x-3)}-\dfrac{2(x^2+x-12)}{(x^2+x-12)(x-3)}$ Two separate terms

$\dfrac{3x+5-2x+4}{(x+4)(x-3)}$ Single term, invisible bracket

$\dfrac{(3x+5)}{(x^2+x-12)(x-3)}-\dfrac{2(x^2+x-12)}{(x^2+x-12)(x-3)}$ Separate terms, only one numerator modified

A1 Correct un simplified answer $\dfrac{x-3}{(x+4)(x-3)}$

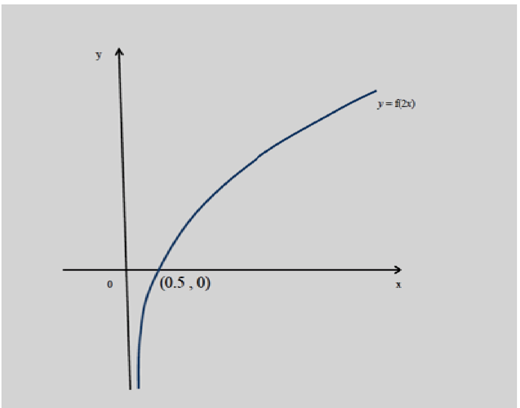
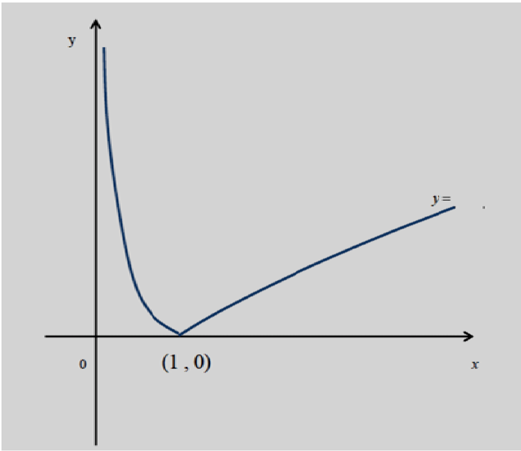
If $\dfrac{x^2-6x-9}{(x^2+x-12)(x-3)}$ scored M1 the fraction must be subsequently be reduced to a correct $\dfrac{x-3}{x^2+x-12}$ or $\dfrac{(x-3)(x-3)}{(x+4)(x-3)(x-3)}$ to score this mark.

A1 cao $\dfrac{1}{(x+4)}$

Do Not isw in this question.

The method of partial fractions is perfectly acceptable and can score full marks

$$\underbrace{\frac{3x+5}{(x+4)(x-3)}}_{B1}-\frac{2}{x-3}=\underbrace{\frac{1}{x+4}+\frac{2}{x-3}}_{M1A1}-\frac{2}{x-3}=\underbrace{\frac{1}{x+4}}_{A1}$$

Question Number	Scheme		Marks
2.(a)		Shape	B1
		(0.5, 0)	B1
			(2)
(b)		Shape	B1
		(1,0)	B1
		Cusp at (1,0)	B1
			(3)
			(5 marks)

Notes for Question 2

(a)

B1 Award for the correct shape. Look for an increasing function with decreasing gradient. Condone linear looking functions in the first quadrant. It needs to look asymptotic at the *y* axis and have no obvious maximum point. It must be wholly contained in quadrants 1 and 4
See practice and qualification items for clarification.

B1 Crosses *x* axis at $\left(\frac{1}{2},0\right)$. Accept $\frac{1}{2}$, 0.5 or even $\left(0,\frac{1}{2}\right)$ marked on the correct axis.
There must be a graph for this mark to be scored.

(b)

B1 Correct shape wholly contained in quadrant 1.
The shape to the rhs of the cusp must not have an obvious maximum.
Accept linear, or positive with decreasing gradient. The gradient of the curve to the lhs of the cusp/minimum should always be negative. The curve in this section should not 'bend' back past (1, 0) forming a 'C' shape or have incorrect curvature.
See practice and qualification for clarification.

B1 The curve touches or crosses the *x* axis at (1, 0). Allow for the curve passing through a point marked '1' on the *x* axis. Condone the point marked on the correct axis as (0, 1)

B1 Award for a cusp, not a minimum at (1,0)

Note that $\mathrm{f}\left(|x|\right)$ scores B0 B1 B0 under the scheme.

If the graphs are not labelled (a) and (b), then they are to be marked in the order they are presented

Question Number	Scheme	Marks
3.(a)	$7\cos x + \sin x = R\cos(x-\alpha)$	
	$R = \sqrt{(7^2+1^2)} = \sqrt{50} = (5\sqrt{2})$	B1
	$\alpha = \arctan\left(\frac{1}{7}\right) = 8.13... = \text{awrt } 8.1^0$	M1A1
		(3)
(b)	$\sqrt{50}\cos(x-8.1) = 5 \Rightarrow \cos(x-8.1) = \frac{5}{\sqrt{50}}$	M1
	$x-8.1 = 45 \Rightarrow x = 53.1^0$	M1,A1
	AND $x-8.1 = 315 \Rightarrow x = 323.1^0$	M1A1
		(5)
(c)	One solution if $\frac{k}{\sqrt{50}} = \pm 1, \Rightarrow k = \pm\sqrt{50}$ ft on R	M1A1ft
		(2)
		(10 marks)

Notes for Question 3

(a)

B1 $R=\sqrt{50}$. Accept $5\sqrt{2}$ Accept $R=\pm\sqrt{50}$

Do not accept $R=\sqrt{(7^2+1^2)}$ or the decimal equivalent 7.07…unless you see $\sqrt{50}$ or $5\sqrt{2}$ as well

M1 For $\tan\alpha=\pm\frac{1}{7}$ or $\tan\alpha=\pm\frac{7}{1}$. Condone if this comes from $\cos\alpha=7, \sin\alpha=1$

If R is used then only accept $\sin\alpha=\pm\frac{1}{R}$ or $\cos\alpha=\pm\frac{7}{R}$

A1 $\alpha=$ awrt 8.1.

Be aware that $\tan\alpha=7\Rightarrow\alpha=81.9$ can easily be mistaken for the correct answer

Note that the radian answer awrt $0.1418\ldots$ is A0

(b)

M1 For using their answers to part (a) and moving from $R\cos(x\pm\alpha)=5\Rightarrow\cos(x\pm\alpha)=\frac{5}{R}$ using their numerical values of R and α

This may be implied for sight of 53.1 if R and α were correct

M1 For achieving $x\pm\alpha=$ awrt 45^0 or 315, leading to one value of x **in the range**

Note that for this to be scored R has to be correct (to 2sf) as awrt 45, 315 must be achieved

This may be implied for achieving an answer of either $45+\textit{their }\alpha$ or $315+\textit{their }\alpha$

A1 One correct answer, either awrt 53.1° or 323.1°

M1 For an attempt at finding a secondary value of x **in the range**.

Usually this is an attempt at solving $x-\textit{their }8.1^0=360^0-\textit{their }45^0\Rightarrow x=..$

A1 Both values correct awrt 53.1° and 323.1°.

Withhold this mark if there are extra values in the range.

Ignore extra values outside the range

(c)

M1 For stating that $\frac{k}{\textit{their }R}=1$ OR $\frac{k}{\textit{their }R}=-1$

This may be implied by seeing $k=(\pm)\textit{their }R$

A1ft Both values $k=\pm\sqrt{50}$ oe . Follow through on their numerical R

Answers all in radians. Lose the first time that it appears but demand an accuracy of 2dp.

Part (a) $R=\sqrt{50}$ $\alpha=\textit{awrt }0.14$

Part (b) $x=\textit{awrt }0.927,\ 5.64$. Accuracy must be to 3 sf.

With correct working this would score (a) B1M1A0 (b) M1A1A1M1A1

Mixed degrees and radians refer to the main scheme

Question Number	Scheme	Marks
4.(a)	$f(x) \geqslant 3$	M1A1
		(2)
(b)	An attempt to find $2\|3-4x\|+3$ when $x=1$	M1
	Correct answer $fg(1)=5$	A1
		(2)
(c)	$y=3-4x \Rightarrow 4x=3-y \Rightarrow x=\dfrac{3-y}{4}$	M1
	$g^{-1}(x)=\dfrac{3-x}{4}$	A1
		(2)
(d)	$\left[g(x)\right]^2=(3-4x)^2$	B1
	$gg(x)=3-4(3-4x)$	M1
	$gg(x)+\left[g(x)\right]^2=0 \Rightarrow -9+16x+9-24x+16x^2=0$	
	$16x^2-8x=0$	A1
	$8x(2x-1)=0 \Rightarrow x=0, 0.5$ oe	M1A1
		(5)
		(11 marks)

Notes for Question 4

(a)

M1 Attempt at calculating f at x=0. Sight of 3 is sufficient. Accept $\mathrm{f}(x) > 3$ and $x > 3$ for M1,

A1 $\mathrm{f}(x) \geqslant 3$. Accept $y \geqslant 3$, range$\geqslant 3$, $[3, \infty)$

Do not accept $\mathrm{f}(x) > 3$, $x \geqslant 3$

The correct answer is sufficient for both marks.

(b)

M1 A full method of finding fg(1). The order of substituting into the expressions must be correct and $2|x|+3$ must be used as opposed to $2x+3$

Accept an attempt to calculate $2|x|+3$ when x= -1.

Accept an attempt to put x=1 into $3-4x$ and then substituting their answer to $3-4x|_{x=1}$ into $2|x|+3$

Do not accept the substitution of x=1 into $2|x|+3$, followed by their result into '3-4x'

This is evidence of incorrect order.

A1 fg(1)=5.

Watch for $1 \xrightarrow{3-4x} 1 \xrightarrow{2|x|+3} 5$ **which is M1A0**

(c)

M1 Award for an attempt to make x or a swapped y the subject of the formula. It must be a full method and cannot finish $4x = ..$

You can condone at most one 'arithmetic' error for this method mark.

$y = 3-4x \Rightarrow 4x = 3+y \Rightarrow x = \dfrac{3+y}{4}$ is fine for the M1 as there is only one error

$y = 3-4x \Rightarrow 4x = 3-y \Rightarrow x = \dfrac{3}{4} - y$ is fine for the M1 as there is only one error

$y = 3-4x \Rightarrow 4x = 3+y \Rightarrow x = \dfrac{3}{4} + y$ is M0 as there are two arithmetic errors

A1 Obtaining a correct expression $\mathrm{g}^{-1}(x) = \dfrac{3-x}{4}$ oe such as $\mathrm{g}^{-1}(x) = \dfrac{x-3}{-4}$, $\mathrm{g}^{-1}(x) = \dfrac{3}{4} - \dfrac{x}{4}$

It must be in terms of x, but could be expressed 'y=' or $g^{-1}(x) \rightarrow$

(d)

B1 Sight of $[g(x)]^2 = (3-4x)^2$. If only the expanded version appears it must be correct

M1 A full attempt to find $\mathrm{gg}(x) = 3-4(3-4x)$

Condone invisible brackets. Note that it may appear in an equation

A1 $16x^2 - 8x = 0$ Accept other alternatives such as $2x^2 = x$

M1 For factorising their quadratic or cancelling their $Ax^2 = Bx$ by x to get ≥ 1 value of x

If they have a 3TQ then usual methods are applicable.

A1 Both values correct $x = 0, 0.5$ oe

Question Number	Scheme	Marks
5.(a)	$\frac{d}{dx}(\cos 2x) = -2\sin 2x$	B1
	Applies $\frac{vu'-uv'}{v^2}$ to $\frac{\cos 2x}{\sqrt{x}}$ $\quad = \frac{\sqrt{x}\times -2\sin 2x - \cos 2x \times \frac{1}{2}x^{-\frac{1}{2}}}{(\sqrt{x}\,)^2}$	M1A1
	$= \frac{-2\sqrt{x}\sin 2x - \frac{1}{2}x^{-\frac{1}{2}}\cos 2x}{x}$	
		(3)
(b)	$\frac{d}{dx}(\sec^2 3x) = 2\sec 3x \times 3\sec 3x \tan 3x\,(= 6\sec^2 3x \tan 3x\,)$	M1
	$= 6(1+\tan^2 3x)\tan 3x$	dM1
	$= 6(\tan 3x + \tan^3 3x)$	A1
		(3)
(c)	$x = 2\sin\left(\frac{y}{3}\right) \Rightarrow \frac{dx}{dy} = \frac{2}{3}\cos\left(\frac{y}{3}\right)$	M1A1
	$\frac{dy}{dx} = \frac{1}{\frac{2}{3}\cos\left(\frac{y}{3}\right)} = \frac{1}{\frac{2}{3}\sqrt{\left(1-\sin^2\left(\frac{y}{3}\right)\right)}} = \frac{1}{\frac{2}{3}\sqrt{1-\left(\frac{x}{2}\right)^2}}$	dM1
	$\frac{dy}{dx} = \frac{3}{\sqrt{4-x^2}}$ cao	A1
		(4)
		(10 marks)
Alt 5(c)	$y = 3\arcsin\left(\frac{x}{2}\right) \Rightarrow \frac{dy}{dx} = \frac{3}{\sqrt{1-\left(\frac{x}{2}\right)^2}} \times \frac{1}{2}$	M1dM1A1
	$\frac{dy}{dx} = \frac{3}{\sqrt{4-x^2}}$	A1
	M1 Rearranging to $y = A\arcsin Bx$ and differentiating to $\frac{dy}{dx} = \frac{A}{\sqrt{1-Bx^2}}$	
	dM1 As above, but form of the rhs must be correct $\frac{dy}{dx} = \frac{C}{\sqrt{1-\left(\frac{x}{2}\right)^2}}$	**(4)**
	A1 Correct but un simplified answer	

Notes for Question 5

(a)

B1 Award for the sight of $\frac{\mathrm{d}}{\mathrm{d}x}(\cos 2x) = -2\sin 2x$. This could be seen in their differential.

M1 Applies $\frac{vu'-uv'}{v^2}$ to $\frac{\cos 2x}{\sqrt{x}}$

If the rule is quoted it must be correct. There must have been some attempt to differentiate both terms. If the rule is not quoted (nor implied by their working, with terms written out u=…,u'=….,v=….,v'=….followed by their $\frac{vu'-uv'}{v^2}$) then only accept answers of the form

$$\frac{\sqrt{x}\times \pm A\sin 2x - \cos 2x \times Bx^{-\frac{1}{2}}}{(\sqrt{x})^2 \text{ or } x^{\frac{1}{4}}}$$

A1 Award for a correct answer. This does not need to be simplified.

Alt (a) using the product rule

B1 Award for the sight of $\frac{\mathrm{d}}{\mathrm{d}x}(\cos 2x) = -2\sin 2x$. This could be seen in their differential.

M1 Applies $vu'+uv'$ to $x^{-\frac{1}{2}}\cos 2x$. If the rule is quoted it must be correct. There must have been some attempt to differentiate both terms. If the rule is not quoted (nor implied by their working, with terms written out u=…,u'=….,v=….,v'=….followed by their $vu'+uv'$) then only accept answers of the form

$$\pm Ax^{-\frac{1}{2}}\sin 2x - Bx^{-\frac{3}{2}}\cos 2x$$

A1 Award for a correct answer. This does not need to be simplified.

$$-2x^{-\frac{1}{2}}\sin 2x - \frac{1}{2}x^{-\frac{3}{2}}\cos 2x$$

(b)

M1 Award for a correct application of the chain rule on $\sec^2 3x$
Sight of $C\sec 3x \sec 3x \tan 3x$ is sufficient

dM1 Replacing $\sec^2 3x = 1+\tan^2 3x$ in their derivative to create an expression in just $\tan 3x$. It is dependent upon the first M being scored.

A1 The correct answer $6(\tan 3x + \tan^3 3x)$. There is no need to write $\mu = 6$

Alt (b) using the product rule

M1 Writes $\sec^2 3x$ as $\sec 3x \times \sec 3x$ and uses the product rule with $u' = A\sec 3x \tan 3x$ and $v' = B\sec 3x \tan 3x$ to produce a derivative of the form $A\sec 3x \tan 3x \sec 3x + B\sec 3x \tan 3x \sec 3x$

dM1 Replaces $\sec^2 3x$ with $1+\tan^2 3x$ to produce an expression in just $\tan 3x$. It is dependent upon the first M being scored.

Notes for Question 5 Continued

A1 The correct answer $6(\tan 3x + \tan^3 3x)$. There is no need to write $\mu = 6$

Alt (b) using $\sec 3x = \dfrac{1}{\cos 3x}$ **and proceeding by the chain or quotient rule**

M1 Writes $\sec^2 3x$ as $(\cos 3x)^{-2}$ and differentiates to $A(\cos 3x)^{-3}\sin 3x$

Alternatively writes $\sec^2 3x$ as $\dfrac{1}{(\cos 3x)^2}$ and achieves $\dfrac{(\cos 3x)^2 \times 0 - 1 \times A\cos 3x \sin 3x}{\left(\cos^2 3x\right)^2}$

dM1 Uses $\dfrac{\sin 3x}{\cos 3x} = \tan 3x$ and $\dfrac{1}{\cos^2 3x} = \sec^2 3x$ and $\sec^2 3x = 1 + \tan^2 3x$ in their derivative to create an expression in just $\tan 3x$. It is dependent upon the first M being scored.

A1 The correct answer $6(\tan 3x + \tan^3 3x)$. There is no need to write $\mu = 6$

Alt (b) using $\sec^2 3x = 1 + \tan^2 3x$

M1 Writes $\sec^2 3x$ as $1 + \tan^2 3x$ and
uses chain rule to produce a derivative of the form $A\tan 3x \sec^2 3x$
or the product rule to produce a derivative of the form $C\tan 3x \sec^2 3x + D\tan 3x \sec^2 3x$

dM1 Replaces $\sec^2 3x = 1 + \tan^2 3x$ to produce an expression in just $\tan 3x$. It is dependent upon the first M being scored.

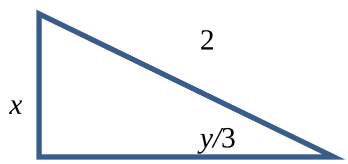
A1 The correct answer $6(\tan 3x + \tan^3 3x)$. There is no need to write $\mu = 6$

(c)

M1 Award for knowing the method that $\sin\left(\dfrac{y}{3}\right)$ differentiates to $\cos\left(\dfrac{y}{3}\right)$ The lhs does not need to be correct/present. Award for $2\sin\left(\dfrac{y}{3}\right) \to A\cos\left(\dfrac{y}{3}\right)$

A1 $x = 2\sin\left(\dfrac{y}{3}\right) \Rightarrow \dfrac{dx}{dy} = \dfrac{2}{3}\cos\left(\dfrac{y}{3}\right)$. Both sides must be correct

dM1 Award for inverting their $\dfrac{dx}{dy}$ and using $\sin^2\dfrac{y}{3} + \cos^2\dfrac{y}{3} = 1$ to produce an expression for $\dfrac{dy}{dx}$ in terms of x only. It is dependent upon the first M 1 being scored.
An alternative to Pythagoras is a triangle.



$$\sin\left(\frac{y}{3}\right) = \frac{x}{2} \Rightarrow \cos\left(\frac{y}{3}\right) = \frac{\sqrt{4 - x^2}}{2}$$

Notes for Question 5 Continued
Candidates who write $\dfrac{\mathrm{d}y}{\mathrm{d}x}=\dfrac{3}{2\cos\left(\arcsin\left(\dfrac{x}{2}\right)\right)}$ do not score the mark. BUT $\dfrac{\mathrm{d}y}{\mathrm{d}x}=\dfrac{3}{2\sqrt{1-\sin^2\left(\arcsin\left(\dfrac{x}{2}\right)\right)}}$ does score M1 as they clearly use a correct Pythagorean identity as required by the notes. A1 $\dfrac{\mathrm{d}y}{\mathrm{d}x}=\dfrac{3}{\sqrt{4-x^2}}$. Expression must be in its simplest form. Do not accept $\dfrac{\mathrm{d}y}{\mathrm{d}x}=\dfrac{3}{2\sqrt{1-\frac{1}{4}x^2}}$ or $\dfrac{\mathrm{d}y}{\mathrm{d}x}=\dfrac{1}{\frac{1}{3}\sqrt{4-x^2}}$ for the final A1

Question Number	Scheme	Marks
6.(i)	$\text{cosec}2x = \dfrac{1}{\sin 2x}$	M1
	$= \dfrac{1}{2\sin x \cos x}$	M1
	$= \dfrac{1}{2}\text{cosec}x \sec x \Rightarrow \quad \lambda = \dfrac{1}{2}$	A1
		(3)
(ii)	$3\sec^2\theta + 3\sec\theta = 2\tan^2\theta \Rightarrow 3\sec^2\theta + 3\sec\theta = 2(\sec^2\theta - 1)$	M1
	$\sec^2\theta + 3\sec\theta + 2 = 0$	
	$(\sec\theta + 2)(\sec\theta + 1) = 0$	M1
	$\sec\theta = -2, -1$	A1
	$\cos\theta = -0.5, -1$	M1
	$\theta = \dfrac{2\pi}{3}, \dfrac{4\pi}{3}, \pi$	A1A1
		(6)
		(9 marks)
ALT (ii)	$3\sec^2\theta + 3\sec\theta = 2\tan^2\theta \Rightarrow 3\times\dfrac{1}{\cos^2\theta} + 3\times\dfrac{1}{\cos\theta} = 2\times\dfrac{\sin^2\theta}{\cos^2\theta}$	
	$3 + 3\cos\theta = 2\sin^2\theta$	
	$3 + 3\cos\theta = 2(1 - \cos^2\theta)$	M1
	$2\cos^2\theta + 3\cos\theta + 1 = 0$	
	$(2\cos\theta + 1)(\cos\theta + 1) = 0 \Rightarrow \cos\theta = -0.5, -1$	M1A1
	$\theta = \dfrac{2\pi}{3}, \dfrac{4\pi}{3}, \pi$	M1,A1,A1
		(6)
		(9 marks)

Notes for Question 6

(i)

M1 Uses the identity $\operatorname{cosec}2x = \dfrac{1}{\sin 2x}$

M1 Uses the correct identity for $\sin 2x = 2\sin x\cos x$ in their expression.
Accept $\sin 2x = \sin x\cos x + \cos x\sin x$

A1 $\lambda = \dfrac{1}{2}$ following correct working

(ii)

M1 Replaces $\tan^2\theta$ by $\pm\sec^2\theta \pm 1$ to produce an equation in just $\sec\theta$

M1 Award for a forming a 3TQ=0 in $\sec\theta$ and applying a correct method for factorising, or using the formula, or completing the square to find two answers to $\sec\theta$

If they replace $\sec\theta = \dfrac{1}{\cos\theta}$ it is for forming a 3TQ in $\cos\theta$ and applying a correct method for finding two answers to $\cos\theta$

A1 Correct answers to $\sec\theta = -2, -1$ or $\cos\theta = -\dfrac{1}{2}, -1$

M1 Award for using the identity $\sec\theta = \dfrac{1}{\cos\theta}$ and proceeding to find at least one value for θ.
If the 3TQ was in cosine then it is for finding at least one value of θ.

A1 Two correct values of θ. All method marks must have been scored.
Accept two of $120°, 180°, 240°$ or two of $\dfrac{2\pi}{3}, \dfrac{4\pi}{3}, \pi$ or two of awrt 2dp 2.09, 3.14, 4.19

A1 All three answers correct. They must be given in terms of π as stated in the question.
Accept $0.\dot{6}\pi, 1.\dot{3}\pi, \pi$
Withhold this mark if further values in the range are given. All method marks must have been scored.
Ignore any answers outside the range.

Alt (ii)

M1 Award for replacing $\sec^2\theta$ with $\dfrac{1}{\cos^2\theta}$, $\sec\theta$ with $\dfrac{1}{\cos\theta}$, $\tan^2\theta$ with $\dfrac{\sin^2\theta}{\cos^2\theta}$ multiplying through by $\cos^2\theta$ (seen in at least 2 terms) and replacing $\sin^2\theta$ with $\pm 1 \pm \cos^2\theta$ to produce an equation in just $\cos\theta$

M1 Award for a forming a 3TQ=0 in $\cos\theta$ and applying a correct method for factorising, or using the formula, or completing the square to find two answers to $\cos\theta$

A1 $\cos\theta = -\dfrac{1}{2}, -1$

M1 Proceeding to finding at least one value of θ from an equation in $\cos\theta$.

A1 Two correct values of θ. All method marks must have been scored
Accept two of $120°, 180°, 240°$ or two of $\dfrac{2\pi}{3}, \dfrac{4\pi}{3}, \pi$ or two of awrt 2dp 2.09, 3.14, 4.19

A1 All three answers correct. They must be given in terms of π as stated in the question.

Notes for Question 6 Continued
Accept $0.\dot{6}\pi, 1.\dot{3}\pi, \pi$ All method marks must have been scored. Withhold this mark if further values in the range are given. Ignore any answers outside the range

Question Number	Scheme	Marks
7.(a)	$f(x)=0 \Rightarrow x^2+3x+1=0$	
	$\Rightarrow x=\frac{-3\pm\sqrt{5}}{2}=$ awrt -0.382, -2.618	M1A1
		(2)
(b)	Uses $vu'+uv'$ $f'(x)=e^{x^2}(2x+3)+(x^2+3x+1)e^{x^2}\times 2x$	M1A1A1
		(3)
(c)	$e^{x^2}(2x+3)+(x^2+3x+1)e^{x^2}\times 2x=0$	
	$\Rightarrow e^{x^2}\left\{2x^3+6x^2+4x+3\right\}=0$	M1
	$\Rightarrow x(2x^2+4)=-3(2x^2+1)$	M1
	$\Rightarrow x=-\frac{3(2x^2+1)}{2(x^2+2)}$	A1*
		(3)
(d)	Sub $x_0=-2.4$ into $x_{n+1}=-\frac{3(2x_n{}^2+1)}{2(x_n{}^2+2)}$	
	$x_1=awrt-2.420,\ x_2=awrt-2.427\ \ x_3=awrt-2.430$	M1A1,A1
		(3)
(e)	Sub x= - 2.425 and -2.435 into $f'(x)$ and start to compare signs	
	$f'(-2.425)=+22.4, f'(-2.435)=-15.02$	**M1**
	Change in sign, hence $f'(x)=0$ in between. Therefore α = - 2.43 (2dp)	A1
		(2)
		(13 marks)
Alt 7.(c)	$x=-\frac{3(2x^2+1)}{2(x^2+2)}$ $\Rightarrow 2x(x^2+2)=-3(2x^2+1)$ $\Rightarrow 2x^3+6x^2+4x+3=0$	M1
	$f'(x)=e^{x^2}\left\{2x^3+6x^2+4x+3\right\}=0$ when $2x^3+6x^2+4x+3=0$	M1
	Hence the minimum point occurs when $x=-\frac{3(2x^2+1)}{(2x^2+4)}$	**A1**

Question Number	Scheme	Marks
Alt 1 7(e)	Sub x= - 2.425 and -2.435 into cubic part of $\text{f}'(x) = 2x^3+6x^2+4x+3$ and start to compare signs	
	Adapted $\text{f}'(-2.425)=+0.06, \text{f}'(-2.435)=-0.04$	**M1**
	Change in sign, hence $\text{f}'(x)=0$ in between. Therefore α = - 2.43 (2dp)	A1
		(2)
Alt 2 7 (e)	Sub x= - 2.425, -2.43 and -2.435 into $\text{f}(x)=(x^2+3x+1)e^{x^2}$ and start to compare sizes	
	$\text{f}(-2.425)=-141.2, \text{f}(-2.435)=-141.2, \text{f}(-2.43)=-141.3$	**M1**
	$\text{f}(-2.43)<\text{f}(-2.425), \text{f}(-2.43)<\text{f}(-2.435)$. Therefore α = - 2.43 (2dp)	A1
		(2)

Notes for Question 7

(a)

M1 Solves $x^2+3x+1=0$ by completing the square or the formula, producing two 'non integer answers. **Do not accept factorisation here** . Accept awrt -0.4 and -2.6 for this mark

A1 Answers correct. Accept awrt -0.382, -2.618.

Accept just the answers for both marks. Don't withhold the marks for incorrect labelling.

(b)

M1 Applies the product rule $vu'+uv'$ to $(x^2+3x+1)e^{x^2}$.

If the rule is quoted it must be correct and there must have been some attempt to differentiate both terms. If the rule is not quoted (nor implied by their working, ie. terms are written out u=…,u'=….,v=….,v'=….followed by their vu'+uv') only accept answers of the form

$$\left(\frac{\text{d}y}{\text{d}x}\right)=\text{f}'(x)=e^{x^2}(Ax+B)+(x^2+3x+1)Cxe^{x^2}$$

A1 One term of $\text{f}'(x)=e^{x^2}(2x+3)+(x^2+3x+1)e^{x^2}\times 2x$ correct.

There is no need to simplify

A1 A fully correct (un simplified) answer $\text{f}'(x)=e^{x^2}(2x+3)+(x^2+3x+1)e^{x^2}\times 2x$

(c)

M1 Sets their $\text{f}'(x)=0$ and either factorises out, or cancels by e^{x^2} to produce a polynomial equation in x

M1 Rearranges the cubic polynomial to $Ax^3+Bx=Cx^2+D$ and factorises to reach $x(Ax^2+B)=Cx^2+D$ or equivalent

A1* Correctly proceeds to $x=-\dfrac{3(2x^2+1)}{2(x^2+2)}$. This is a given answer

Notes on Question 7 Continued

(c) Alternative to (c) working backwards

M1 Moves correctly from $x=-\frac{3(2x^2+1)}{2(x^2+2)}$ to $2x^3+6x^2+4x+3=0$

M1 States or implies that $f'(x)=0$

A1 Makes a conclusion to tie up the argument

For example, hence the minimum point occurs when $x=-\frac{3(2x^2+1)}{(2x^2+4)}$

(d)

M1 Sub $x_0=-2.4$ into $x_{n+1}=-\frac{3(2x_n{}^2+1)}{2(x_n{}^2+2)}$

This may be implied by awrt -2.42, or $x_{n+1}=-\frac{3(2\times-2.4^2+1)}{2(-2.4^2+2)}$

A1 Awrt. $x_1=-2.420$.

The subscript is not important. Mark as the first value given

A1 awrt $x_2=-2.427$ awrt $x_3=-2.430$

The subscripts are not important. Mark as the second and third values given

(e)

Note that continued iteration is not allowed

M1 Sub *x*= - 2.425 and -2.435 into $f'(x)$, starts to compare signs and gets at least one correct to 1 sf rounded or truncated.

A1 Both values correct (1sf rounded or truncated), a reason and a minimal conclusion

Acceptable reasons are change in sign, positive and negative and $f'(a)\times f'(b)<0$

Minimal conclusions are hence $\alpha=-2.43$, hence shown, hence root

Alt 1 using adapted $f'(x)$

(e)

M1 Sub *x*= - 2.425 and -2.435 into cubic part of $f'(x)$, starts to compare signs and gets at least one correct to 1 sf rounded or truncated.

A1 Both values correct of adapted $f'(x)$ correct (1sf rounded or truncated), a reason and a minimal conclusion

Acceptable reasons are change in sign, positive and negative and $f'(a)\times f'(b)<0$

Minimal conclusions are hence $\alpha=-2.43$, hence shown, hence root

Alt 2 using $f(x)$

(e)

M1 Sub *x*= - 2.425, -2.43 and -2.435 into $f(x)$, starts to compare sizes and gets at least one correct to 4sf rounded

A1 All three values correct of $f(x)$ correct (4sf rounded), a reason and a minimal conclusion

Acceptable reasons are $f(-2.43)<f(-2.425)$, $f(-2.43)<f(-2.435)$, a sketch

Minimal conclusions are hence $\alpha=-2.43$, hence shown, hence root

Question Number	Scheme	Marks
8.(a)	$t=0 \Rightarrow P=\frac{8000}{1+7}=1000$ cao	M1A1
		(2)
(b)	$t \to \infty \quad P \to \frac{8000}{1}=8000$	B1
		(1)
(c)	$t=3,\ P=2500 \Rightarrow 2500=\frac{8000}{1+7e^{-3k}}$	B1
	$e^{-3k}=\frac{2.2}{7}=(0.31..)$ oe	M1,A1
	$k=-\frac{1}{3}\ln\left(\frac{2.2}{7}\right)=\text{awrt } 0.386$	M1A1
		(5)
(d)	Sub t=10 into $P=\frac{8000}{1+7e^{-0.386t}} \Rightarrow P=6970$ cao	M1A1
		(2)
(e)	$\frac{dP}{dt}=-\frac{8000}{(1+7e^{-kt})^2}\times -7ke^{-kt}$	M1,A1
	Sub t=10 $\left.\frac{dP}{dt}\right\|_{t=10}=346$	A1
		(3)
		(13 marks)

Notes for Question 8

(a)

M1 Sets t=0, giving $e^{-k\times 0}=1$. Award if candidate attempts $\frac{8000}{1+7\times 1}, \frac{8000}{8}$

A1 Correct answer only 1000. Accept 1000 for both marks as long as no incorrect working is seen.

(b)

B1 8000. Accept $P<8000$. Condone $P \leqslant 8000$ but not $P>8000$

(c)

B1 Sets both $t=3$, and $P=2500 \Rightarrow 2500=\frac{8000}{1+7e^{-3k}}$

This may be implied by a subsequent correct line.

M1 Rearranges the equation to make $e^{\pm 3k}$ the subject. They need to multiply by the $1+7e^{-3k}$ term, and proceed to $e^{\pm 3k}=A, \quad A>0$

A1 The correct intermediate answer of $e^{-3k}=\frac{2.2}{7}, \frac{11}{35}$ or equivalent. Accept awrt 0.31..

Alternatively accept $e^{3k}=\frac{35}{11}, 3.18..$ or equivalent.

M1 Proceeds from $e^{\pm 3k}=A, \quad A>0$ by correctly taking *ln*'s and then making *k* the subject of the formula.

Award for $e^{-3k}=A \Rightarrow -3k=\ln(A) \Rightarrow k=\frac{\ln(A)}{-3}$

If e^{3k} was found accept $e^{3k}=C \Rightarrow 3k=\ln C \Rightarrow k=\frac{\ln C}{3}$ As with method 1, C >0

A1 Awrt $k=0.386$ 3dp

(d)

M1 Substitutes t=10 into $P=\frac{8000}{1+7e^{-kt}}$ with their numerical value of *k* to find *P*

A1 $(P=)\,6970$ or other exact equivalents like 6.97×10^{3}

(e)

M1 Differentiates using the chain rule to a form $\frac{\mathrm{d}P}{\mathrm{d}t}=\frac{C}{(1+7e^{-kt})^{2}}\times e^{-kt}$

Accept an application of the quotient rule to achieve $\frac{(1+7e^{-kt})\times 0-C\times -e^{-kt}}{(1+7e^{-kt})^{2}}$

A1 A correct un simplified $\frac{\mathrm{d}P}{\mathrm{d}t}=-\frac{8000}{(1+7e^{-kt})^{2}}\times -7ke^{-kt}$.

The derivative can be given in terms of *k*. If a numerical value is used you may follow through on incorrect values.

A1 Awrt 346. Note that M1 must have been achieved. Just the answer scores 0

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